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NOTES ON THE TEACHING OF MATHEMATICS IN AN OBERREALSCHULE.

BY M. O. TRIPP.

The *Realschulen* of Germany originated in a reactionary movement against the humanistic studies. One fundamental idea in the starting of the *Realschulen* was preparation for commercial professions, while the humanistic institutions prepared for the learned ones. In the course of time, however, the *Realschulen* have enlarged their curriculum to meet the demands of general culture.

The *Oberrealschule* with its wide range of subjects gives a course of nine years, the same length as that of the humanistic *Gymnasium*. Between these two classes of schools there is the *Realgymnasium,* with Latin but no Greek, and having a nine years' course also. Girls are prepared for the university in *Lyceen* which give a three years' course following graduation from the *Höhere Mädchenschulen.*

Pupils are generally at least nine years of age when they enter the lowest class in the *Oberrealschule*. They must possess the following qualifications: ability to read German or Latin print, a legible handwriting in either German or Latin script, readiness in taking down simple dictation, ability to carry out the four fundamental operations of arithmetic, and some knowledge of biblical history. The subjects taught in the *Oberrealschule* are religion, German, French, English, history, geography, mathematics, natural science, drawing, penmanship, singing and gymnastics. The average age of graduation is about twenty years. Graduates from the *Oberrealschulen* and *Gymnasien* are granted admission to German universities on practically the same basis. In consequence of government control and centralization of authority there is almost absolute uniformity in the courses of study, so that when an observer has visited one school of a particular kind the work may be considered as typical of all schools of that class.

The notes which form the basis of this article were taken at the *Kaiser Wilhelm II Oberrealschule* in Göttingen where I carefully observed the methods of instruction in various classes, after having obtained special permission from the Prussian Minister of Ecclesiastical, Instructional and Medicinal Affairs. The work in mathematics is placed under the heading ***Rechnen und Mathematik*** and covers a wide variety of subjects from elementary arithmetic through analytic geometry to a little calculus and descriptive geometry. In mathematics and physical science the *Oberrealschule* gives a number of hours greater than any other secondary school. More time is devoted to mathematics than to any other subject in the curriculum, French being a close second. Religion and drawing each take up about one-third as much time. Mathematics which is taught four hours per week for the first two years, six hours per week for the next three years, and five hours a week for the last four years occupies a trifle more than one-sixth of the entire time. The total time of instruction in all subjects varies from thirty to thirty-eight hours (*i. e.,* fifty-minute periods with ten minutes intermission) per week. Such a thing as a study period is conspicuously absent. It is quite common to have two subjects taught simultaneously and even during the same period. In one class half the period was given to descriptive geometry and the other half to calculus. The length of vacation is about the same as in the secondary schools of this country, but recitations are held six days in the week, so that more time is really given to school work.

Perhaps the most striking feature of the mathematical work is the limited use of blackboards. In the recitation rooms there was usually a small blackboard where only one person could work at a time, and even then much work was carried on orally which in American schools would have been illustrated at the board. In a long discussion of elementary processes in algebra, students of the *Untertertia* (corresponding approximately to our eighth grade) showed unusual facility in handling orally algebraic expressions, such as those ininvolved in factoring $a^6 — b^6$ and $a^5 — b^5$. The discussion of powers of 10 ran up as high as quadrillions. The only blackboard work during the recitation was an application of the simple equation to the evaluation of a circulating decimal. In another class logarithms were discussed without using the blackboard. In geometry and trigonometry a figure was usually drawn on the board and then the whole class took part in the discussion of the theorem or problem. I did not see any demonstrations written on the blackboard. The teacher questions continually, giving help whenever necessary to secure prompt and complete statements in reply. A large amount of time is spent in reviewing previous theorems. In talking with the instructors I found that they did not want any more blackboard space in their recitation rooms, since they believe that the mental training developed with one small blackboard and much oral discussion is greater than when there is a large amount of written work.

Text-books play a rather unimportant part in the mathematical instruction, although students are required to have them. The instructors do not follow a text closely and, in general, pay no attention to it except for exercises. One instructor in analytic geometry, however, told me that occasionally he had his students study the text carefully. I found the class in trigonometry using a text by Holzmüller and tables by Schlömilch. In one class I saw a copy of Bardey's *Aufgabensammlung*. The Kambly texts are more widely used in Germany than those of any other author. They differ from our American textbooks, since the elaboration of detail is left to the individual teachers, and the subject is given in outline; *e. g.*, the book on *Planimetrie* covers, in this way, what we give in plane geometry, plane trigonometry and also some solid geometry. The aim of the teacher is not to follow the text, but rather to make the text follow his work by way of supplementing, thus furnishing exercises which otherwise would necessitate loss of time in dictation. Notwithstanding the little stress placed on text-book work the instructors do not fall into the habit of lecturing. Instructors are not free to introduce a new text. This can be done only by higher authority.

Home work does not play so important a part as with us. It is generally arranged so that the preparation of lessons at home shall not occupy more than half as much time as the recitation period. In each class only one exercise was given out for home work. The large amount of time devoted to recitations enables the instructors to spend much time in review and in the solution of exercises. The recitation period is also a period for the acquisition of new knowledge and hence this mode of instruction has been aptly termed the study-recitation method.

The method of conducting a recitation, as it prevails at Göttingen, is illustrated by the following lesson in analytic geometry on the parabola.

I. *Discussion of the equation of the parabola.* This equation was written in the form $y^2 = 2px$. The instructor then drew a figure on the board and asked questions about the parameter $2p$, directrix, etc. A student derived the equation of the parabola, explaining at the same time that he worked at the board. During this time the instructor repeatedly asked questions, which were answered as well by students in the seats as by the one at the board. Attention was called to the fact that as x increases y does; also, and by questioning it was developed that to one value of x there are two of y, and to one of y only one of x. At times students were slow in answering.

II. *Condition that a point is (1) inside, (2) outside, (3) on the parabola.*

(1) The instructor placed a point P inside the parabola and proceeded to quiz concerning it, while the students worked at their seats. Meanwhile the instructor placed results on the black-board. In this way there was developed for the point (x, y) within the parabola

$$(x + p/2)^2 - (x - p/2)^2 > y^2$$

or

$$2px > y^2.$$

The instructor stated the result: If a point (x, y) lies inside the parabola then is $2px > y^2$.

(2) The point outside the parabola was considered and the relation

$$(x + p/2)^2 < (x - p/2)^2 + y^2$$

or

$$2px < y^2$$

was developed in the same way as before. This time, however, a student was asked to state the result: If a point (x, y) lies outside a parabola then is $2px < y^2$.

(3) The instructor said "when $y^2 = px$ we are absolutely sure that the point (x, y) lies on the parabola." Attention was called to the fact that the parabola is the geometric locus of $y^2 = 2px$. The three results were then placed together, as a test whether P is in, on or outside the parabola.

The correspondence between the algebraic and geometric considerations of the parabola were discussed. The construction of points of the parabola with ruler and compasses followed. The equation $y^2 = 2px$ was read as a proportion by students and the use of the proportion was explained. One student confessed that he did not understand and was given the required assistance by the instructor.

III. *Computation of areas of parabolic sections cut off by perpendiculars to the axis.* From previous proof it was stated that the area is equal to two thirds of the triangle formed by the perpendicular and the tangents at its extremities. Attention was called to the fact that the subtangent is bisected at the vertex, and then the area was expressed in the form $4/8 x\sqrt{2px}$. The latter expression was then stated in words. The instructor told the students not to memorize this result. He spoke of the irrationality appearing in the expression and emphasized the fact that it is a different kind of irrationality from that of the Ludolphian number.

IV. *Consideration of the area cut off by the latus rectum.* A student was required to give a geometric interpretation of the expression $4/3 \cdot p/2\sqrt{2p \cdot p/2}$ in terms of the latus rectum.

V. *Solution of exercises.* Students were required to solve the following exercise: In a parabola the parameter is 10 cm. How great is the section corresponding to the abscissa $x = 15$? The instructor gave the equation of the parabola and asked about the magnitude of the parameter. Finally an exercise taken from the text, Knop's "Analytic Geometry," was considered. The students did not answer well when asked how to attack the problem, showing that they had not fully comprehended the theory.

It is, of course, true that the plan of teaching with a small blackboard and a text merely to supplement the recitation work produces and requires stronger teachers than an abundance of blackboard work and slavish obedience to the text-book. It is doubtless true that, in this country, teachers often spend too much time in mere routine blackboard work and not enough in careful questioning and discussion. It is highly probable that our better teachers, with an adequate blackboard equipment and a well selected text, do fully as good work as the Germans. There is, in fact, a decided advantage in training students to the use of books so that, to a certain extent, they may become independent.

The success of the Germans is, I believe, due neither to the lack of blackboard work nor to the little stress laid upon text-books; but rather to the highly trained teachers who give instruction in the secondary schools. Ph.D. men are quite common in the *Gymnasien* and *Oberrealschulen*. In the Göttingen *Oberrealschule* eight of the twenty teachers were doctors. I am informed that in some institutions the doctors are in the majority. The status of the teacher with his permanent position and pension is considerably above that of the American teacher. Competition for educational positions is keener than with us because the German teacher, as compared with other members of society, is much better paid than in America.

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